Hjemmeopgavesæt 5

Differentiable Functions of Two Variables

Your answer must be uploaded as a pdf to your DTU Learn account. Remember name and study number in the top of your hand-in. Deadline is 25/2, 23:55.

In your answer you must show in particular that you
- *can characterise domains*
- *can handle level curves and gradients*
- *can parametrize curves and determine tangent vectors*
- *can find stationary points and determine whether they are local extrema*
- *can give geometric interpretations of second-degree equations in x and y*
- *can find global extrema on bounded, closed sets*
- *can use fitting plots for investigation and presentation*
- *write coherently and precisely and can perform simple mathematical reasoning*

Opgave 1 Investigation of a Function of Two Variables

A function f of two variables is given by the expression

$$f(x,y) = \sqrt{x - y^2 + 1}\,.$$

Furthermore we consider the point $P = (1,1)$.

a) Justify that the level curve K_0 of f is a parabola, and write down the domain $D(f)$ of f.

b) A level curve K_c passes through P. Determine the number c and provide a parametric representation of K_c. Create an illustration on which the tangent vector of the parametrization as well as the gradient of f at P is shown. Justify without further computations that the directional derivative of f at P in the direction of the tangent vector equals 0.

c) Justify that f does not have stationary points.

|||| Opgave 2 Taylor Polynomial and Determination of Conic Section

A real function of two real variables is given by

$$f(x,y) = x^2y^2 - 4xy^2 + xy + 4y^2 - 2y + 1\,.$$

The approximating polynomial of the second degree of $f(x,y)$ with its point at development at $(x_0, y_0) = (2,0)$ is in the following denoted by $P_2(x,y)$.

a) Determine $P_2(x,y)$.

b) The equation $P_2(x,y) = 0$ describes a hyperbola on the (x,y) plane. Determine an equation for the hyperbola, and state its centre and semi-axes.

|||| Opgave 3 Extremum Investigations

A function $f : \mathbb{R}^2 \to \mathbb{R}$ is given by the expression

$$f(x,y) = y \cdot \mathrm{e}^{x(1-y^2)}\,.$$

a) Determine all stationary points of f, and investigate whether f has local extremum at those points.

b) Determine the global maximum and the global minimum of f on the set

$$A = \{(x,y) \mid x \in [-1,1] \;\text{ and }\; y \in [-1,1]\}\,,$$

and state the points where they are found.