# Homework set 2

# Systems of Linear Equations

*Deadline: October 15th at 23:55. Your solution to the three problems must be uploaded as one pdf file to your group's DTU Learn module. Remember name and student number in the top of the document.*

*Explanations and rationale are as usual expected in an appropriate amount in regards to methods and computations. In the evaluation the primary focus will be on your ability to*

- *analyse mathematical problems using systems of linear equations*
- *convert a system of equations to its coefficient and augmented matrices*
- *use row operations for GaussJordan elimination*
- *use the rank of matrices when solving systems of linear equations*
- *convert a reduced augmented matrix to a standard parametric form of the solution set*
- *determine the inverse matrix of an invertible matrix*
- *use Maple/Sympy for illustrative plots*
- *write coherently and precisely and perform simple mathematical reasoning*

## Problem 1 Matrix Equations

Two matrices are given by

$$\mathbf{A} = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 4 & -2 \\ 3 & -6 & 3 \end{bmatrix} \text{ and } \mathbf{B} = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 2 & -1 \end{bmatrix}.$$

a) State on standard parametric form the solution sets of the two matrix equations

$$\mathbf{A} \cdot \mathbf{x} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \text{ and } \mathbf{B} \cdot \mathbf{y} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}.$$

*The homework set continues* $\longmapsto$

b) Show that all solutions to the latter matrix equation in question a) also are solutions to the former.

## |||| Problem 2 Geometric Interpretation of Systems of Linear Equations

In a usual Cartesian $(x, y, z)$ coordinate system four planes $\alpha_1$, $\alpha_2$, $\alpha_3$ and $\alpha_4$ are given by the equations:

$$\begin{aligned} \alpha_1 :& \quad -x + 2y + 2z = 6 \\ \alpha_2 :& \quad 2x - y - z = 0 \\ \alpha_3 :& \quad 4x - y - 7z = -2 \\ \alpha_4 :& \quad -x - y + 8z = 3. \end{aligned} \tag{1}$$

a) Show that the four planes $\alpha_1$, $\alpha_2$, $\alpha_3$ and $\alpha_4$ have exactly one point in common by considering the rank of the augmented matrix and then the rank of the coefficient matrix. State this point.

b) The equation for $\alpha_1$ is now replaced by $2x - y - z = 2$, while the other equations are kept unchanged. Solve the new equation system. Explain the result in light of $\rho(\mathbf{T})$ and $\rho(\mathbf{A})$ .

c) The equation for $\alpha_1$ is now replaced by $x - y + 2z = 1$, while the other equations are kept unchanged. Same question as in b).

d) Use the Maple plotting command `implicitplot3d` to illustrate all above scenarios. Click on the plot so that you can rotate the image such that the solution sets are clearly visualised, and explain in your own words how the solution sets are seen on each of the three figures.

*The homework set continues* ⟼

## Problem 3 Systems of Equations Containing Unknown Coefficients

We are given the coefficient matrix to a homogeneous linear equation system:

$$\mathbf{A} = \begin{bmatrix} 1 & -a & -1 \\ 1 & 1 & a \\ a & -1 & 1 \end{bmatrix} \quad \text{where} \quad a \in \mathbb{R}\,. \tag{2}$$

a) Write out for $a = 1$ the corresponding equation system, and compute within $\mathbb{R}^3$ all of its solutions.

b) Compute for all values of $a$ all real solutions to the system of linear equations:

$$\begin{aligned} x_1 - a\,x_2 - x_3 &= 0 \\ x_1 + x_2 + a\,x_3 &= 0 \\ a\,x - x_2 + x_3 &= 0. \end{aligned} \tag{3}$$

c) For which values of $a$ does $\mathbf{A}$ have an inverse matrix? Find the inverse matrix for $a = 1$ using GaussJordan elimination.

*End of problem sheet*