Homework set 1

Complex Numbers and Functions

For the evaluation of this set it is particularly important that you have shown that you
- *master elementary computations with complex numbers*
- *can switch between the rectangular and exponential forms of a complex number*
- *know the definition of an nth-degree polynomial (see Definition 2.1)*
- *know the connection between roots in a polynomial and a factorisation of the polynomial*
- *masker the theorem of descent*
- *know the structure of the complex roods of a polynomial with real coefficients*
- *have insight into the structure of the complex exponential function*
- *write coherently and precisely and can perform simple mathematical reasoning*

This set of problems is to be solved by hand in an essay style with all necessary explanations included. You own individual solution of the set must be uploaded as one pdf file latest september 24, 23:55, to the DTU Learn module for your group.

Problem 1 Polynomials

Two polynomials P and Q are for $z \in \mathbb{C}$ given by

$$P(z) = z^3 - 7z^2 + 41z - 87 \text{ and } Q(z) = (z-3) \cdot (10z^3 - 27z^2 - 10z + 3)\,.$$

Show that $z_0 = 3$ is a root of both P and Q, for one of the polynomials with an algebraic multiplicity of at least 1 and for the other polynomial with an algebraic multiplicity of at least 2.

a) Determine using the theorem of descent as well as formulas for quadratic equations all roots of P and Q.

The homework set continues ⟶

b) A new polynomial R is for $z \in \mathbb{C}$ given by

$$R(z) = P(z) \cdot Q(z)\,.$$

Write R in both the usual form $a_n z^n + a_{n-1} z^{n-1} + \cdots + a_1 z + a_0$ and in fully factorised form.

Problem 2 Rewritings between Rectangular and Exponential Form

Three complex numbers are given in exponential form as follows:

$$A = 5\,\mathrm{e}^{\frac{5\pi}{4}i} \quad , \quad 3\,\mathrm{e}^{\frac{3\pi}{2}i} \quad \text{og} \quad C = \mathrm{e}^{i\,13\pi}\,.$$

a) Compute the absolute value and the principal argument of A, B and C, and determine their rectangular forms.

We are given two complex numbers in rectangular form:

$$D = 2\sqrt{2} + 2\sqrt{2}i \;\text{ and }\; E = -4\sqrt{3} + 4i\,.$$

b) Determine the polar coordinates of D and E. Utilize the exponential forms of the two numbers to easily compute the number $\dfrac{D^{10}}{E^5}$. State the answer first in exponential form and then in rectangular form.

Problem 3 Solving Equations

a) We are given the trigonometric equation

$$\sin(x) = \frac{1}{2}\,,\; x \in \mathbb{R}.$$

State the solutions to the equation that are found in the interval $[-2\pi\,,\,2\pi]$.

b) We are given the binomial equation

$$z^6 = -1\,,\; z \in \mathbb{C}.$$

Compute all solutions to the equation. State the answer first in exponential form and then in rectangular form.

End of problem sheet

c) We are given the exponential equation

$$e^z = 2i\,,\ z \in \mathbb{C}.$$

Compute all solutions to the equation whose imaginary parts are found in the interval $[-2\pi\,,\,5\pi]$.

End of problem sheet