Homework set 7

Flux through Open and Closed Surfaces

Your answer must be uploaded as pdf to your DTU Learn account. Remember name and study number in the top of your hand-in. Deadline is 14/4 at 23:55.
Note: In your answer you must show in particular that you can

- *design fitting parametrizations*
- *determine anti-derivatives for gradient vector fields*
- *compute flux through an open surface*
- *use Gauss' Theorem*
- *use Stokes' Theorem*
- *create illustrating Maple plots*
- *write coherently and precisely and carry out simple mathematical reasoning*

Problem 1 Flux through Open Surface

A cylindrical surface $\mathcal{F}$ is in part determined by its profile curve in the (x, y) plane:

$$\mathcal{L}: \ (x,y) = (u - \sin(u), 1 - \cos(u)) \, , u \in [\, 0 \, , \, 4\pi \,] \, ,$$

and in part by $z \in [\, 0, y \,]$. Furthermore, a vector field $\mathbf{V}$ is given by

$$\mathbf{V}(x,y,z) = (x+z, -y-z, x-y) \, .$$

a) Determine a parametric representation $\mathbf{r}$ for $\mathcal{F}$. (Hint: First, plot $\mathcal{L}$ to get an impression of the cylindrical surface).

b) Compute the flux of $\mathbf{V}$ through the surface $\mathcal{F}$.

c) Show that $\mathbf{V}$ is a gradient field, and determine all anti-derivatives.

|||| Problem 2 Flux through a Closed Surface and Circulation along its Boundary Curve

Note: Your answer to this problem must show all in-between steps.

A vector field in (x, y, z) space is given by

$$\mathbf{V}(x, y, z) = (2x + 2y\,,\, -2x + 2y\,, z^2)\,.$$

a) Compute the divergence and the curl of $\mathbf{V}$.

A solid region $\mathcal{E}$ in (x, y, z) space has the parametric representation

$$\mathbf{r}(u, v, w) = (\,3w\sin(u)\cos(v),\, 2w\sin(u)\sin(v),\, 2w\cos(u)\,)$$

where $u \in [\,0\,,\,\pi\,]\,, v \in [\,-\pi\,,\,\pi\,]\,, w \in [\,0\,,\,1\,]\,.$

b) Compute the flux

$$\int_{\partial\mathcal{E}} \mathbf{V}\cdot\mathbf{n}_{\mathcal{E}}\,\mathrm{d}\mu$$

both directly as well as by use of Gauss' Theorem. Note: $\partial\mathcal{E}$ is thought to be oriented with an outwards-pointing unit normal vector field.

c) Justify that

$$\mathbf{s}(v, w) = \mathbf{r}(\frac{\pi}{2}, v, w)\,,\, v \in [\,-\pi\,,\,\pi\,]\,, w \in [\,0\,,\,1\,]\,.$$

describes a plannar elliptical region in the (x, y) plane in (x, y, z) space. Determine the circulation of $\mathbf{V}$ along the boundary curve of the elliptical region both directly as well as by use of Stokes' Theorem. Note: The boundary curve is to be given an orientation of your choice that must be shown on a sketch.

End of problem sheet