Homework set 6

Line, Surface, and Volume Integrals

Your answer must be uploaded as pdf to your DTU Learn account. Remember name and study number in the top of your hand-in. Deadline is 17/3 at 23:55.
Note: In your answer you must show in particular that you can

- *design fitting parametrizations of simple curves and planar regions*
- *decsribe the Jacobian functions' significance in integration*
- *compute plane and line integrals*
- *design fitting parametrizations of simple surfaces and spatial regions*
- *compute surface integrals and volume integrals*
- *use illutrating Maple plots for your explanations*
- *write coherently and precisely and with simple mathematical reasoning*

Problem 1 Plane and Line Integrals

In the (x, y) plane we are given the function

$$f(x, y) = x + y\,.$$

Let B denote the closed and bounded set of points in the (x, y) plane that is bounded by the two vertical lines $x = 0$ and $x = \frac{\pi}{4}$ as well as by the graphs of $x + \cos(x)$ and $x + \sin(x)$.

a) Provide a parametric representation of B, and determine the corresponding Jacobian function. Compute the area of B.

b) Determine the plane integral of f over B.

Consider the space curve $\mathcal{K}$ that is formed when the curve $\{(x, y)\,|\,y = 2 \text{ and } 0 \leq x \leq 1\}$ is lifted (vertically) up onto the graph of f.

c) Provide a parametric representation of $\mathcal{K}$, and determine the corresponding Jacobian function. Compute the line integral $\int_{\mathcal{K}} x \cdot y \, \mathrm{d}\mu$.

|||| Problem 2 Surface and Volume Integrals

A region $\mathcal{A}$ in the (x, y) plane is given by

$$\mathcal{A} = \left\{ (x, y) \mid -2 \leq x \leq 2 \text{ and } 0 \leq y \leq 4 - x^2 \right\} .$$

Furthermore, a surface $\mathcal{F}$ is given by the part of the graph of the function

$$h(x, y) = 4 - y - x^2$$

that fulfills $y \geq 0$ and $z \geq 0$.

a) Determine a parametric representation of $\mathcal{A}$ and of $\mathcal{F}$, and state the Jacobian function that corresponds to $\mathcal{F}$.

b) Compute the surface integral of the function $f(x, y, z) = \sqrt{12} \cdot y \cdot \sqrt{1 + 2x^2}$ over $\mathcal{F}$.

Let $\mathcal{B}$ denote the closed spatial region located (vertically) between $\mathcal{A}$ and $\mathcal{F}$.

c) Determine a parametric representation of $\mathcal{B}$ and state the Jacobian function that corresponds to $\mathcal{B}$.

d) Compute the volume integral of $\mathcal{B}$.

|||| Problem 3 Surfaces of Revolution and Spatial Regions of Revolution

A profile curve in the (x, z) plane is given by the set of points

$$\{ (x, z) \mid x = \ln(z) , z \in [2 , 4] \} .$$

The profile curve is rotated about the z axis from the angular position $-\frac{\pi}{4}$ in the (x, y) plane to the angular position $\frac{\pi}{4}$ in the (x, y) plane. Hereby a surface of revolution $\mathcal{F}$ is formed.

a) Determine a parametric representation, partly of the profile curve and partly of $\mathcal{F}$. Determine the Jacobian function corresponding to $\mathcal{F}$.

b) Compute the surface integral

$$\int_{\mathcal{F}} \frac{z^2}{\ln(z)} \, d\mu .$$

A profile region is given by the set of points

$$\left\{ (x, z) \,|\, 0 \le x \le 2 \text{ and } \mathrm{e}^x \le z \le \mathrm{e}^2 \right\} .$$

By rotation of the profile region by an angle of 2π about the z axis, a solid of revolution Ω is formed.

c) Determine a parametric representation, partly of the profile region and partly of Ω. Determine the Jacobian function that corresponds to Ω.

d) Compute the volume integral

$$\int_\Omega y^2 + x^2 \, d\mu \, .$$

End of problem sheet