Homework Set 6

Parametrization and Integration

Deadline is 21/3, 23:55. Exercise 1 and 2 shall not be handed in, but answered in Maple TA where they can be changed. Maple TA is open on Inside from Thursday, 18/3, 12:00. Exercise 3 is an essay-exercise and your answer must be uploaded as a pdf-file to: Assignments on your class' Inside account. Remember name and study number at the top of you answer.

In the exercises to be answered in Maple TA it is important that you can
- *can design parametric representations for geometrical objects in the plane and in space*
- *find and use Jacobi functions corresponding to the given parametric representations*
- *use elementary techniques for integration in more variables*
- *find quantities such as volume and mass by integral calculus*
- *use Maple in advanced computation*

In the essay-assignment, you must demonstrate that you
- *can handle appropriate rewriting of hyperbolic functions and their inverse*
- *use both elementary and Maple techniques for integration*
- *can handle composite functions*
- *can determine curve lengths and surface integrals*
- *can handle and illustrate elementary data sets using Maple*
- *write coherently and precisely and conduct simple mathematical reasoning*

Problem 1 Tetrahedron with mass midpoint. Answered in Maple TA

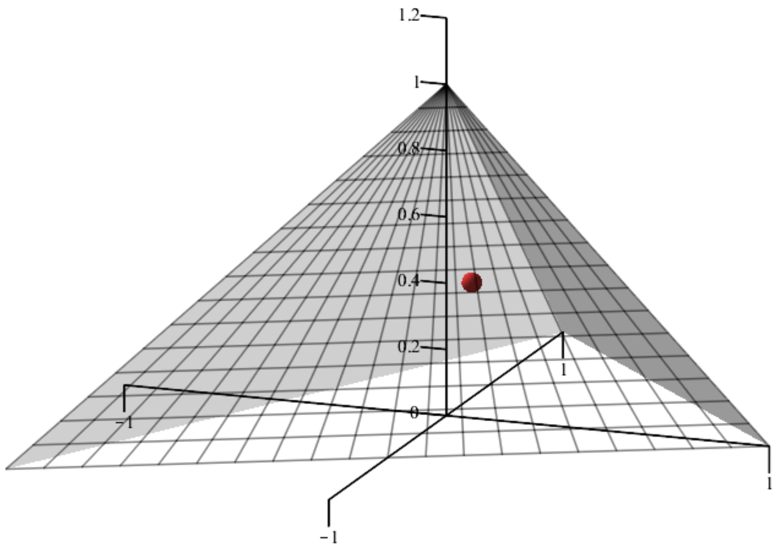
In (x, y, z)-space we consider the four points $A = (0, 1, 0)$, $B = (1, 0, 0)$, $C = (-1, -1, 0)$, and $P = (0, 0, 1)$.

a) Give a parametric representation of the line segment from A to B.

b) Give a parametric representation of the triangle with the vertices A, B and C.

c) Give a parametric representation of the tetrahedron T spanned by A, B, C and P.

The assignment continues ⟼



d) A mass density function $f:\mathbb{R}^3\to\mathbb{R}$ is given by the expression

$$f(x,y,z)=(x+1)\cdot z\,.$$

Determine T's mass and mass midpoint.

|||| Problem 2 Graph surface and solid of revolution. Answered in Maple TA

We consider in the (x,y)-plane a closed and bounded set of points B given by

$$B=\{(x,y)\mid x\in[\,0,\pi\,]\,,\,y\in[\,-x,\,2\sin(x)-x\,]\}\ .$$

Furthermore we consider the function $h:\mathbb{R}^2\to\mathbb{R}$ given by the expression

$$h(x,y)=x+y\,.$$

a) Find a parametric representation for the graph surface

$$\mathcal{F}=\{(x,y,h(x,y))\mid(x,y)\in B\}\ .$$

b) Determine the surface integral $\displaystyle\int_{\mathcal{F}}x\cdot z\,\mathrm{d}\mu\,.$

In (x,z)-plane in the (x,y,z)-space we now consider the profile region

$$A=\{(x,y,z)\mid x\in[\,0\,,\pi\,]\,,\,y=0\,,\,z\in[\,-x\,,\,2\sin(x)-x\,]\}\ .$$

c) How many degrees must A be rotated about the z-axis before the point set – swept by A – reaches the volume 4? Illustrate.

The assignment continues $\longmapsto$

Problem 3 Integral calculus on area functions. Essay-assignment

A curve $K_{\mathbf{r}}$ in the (x, y)-planen is given by the parametric representation

$$\mathbf{r}(t) = \left(\sqrt{2} \cdot \operatorname{arcosh}(t), \sqrt{2} \cdot \operatorname{arsinh}(t) \right), \quad t \in [1, 10] .$$

We interpret $K_{\mathbf{r}}$ as the movement of P' with t as the time parameter.

a) Make a plot of $K_{\mathbf{r}}$ together with 10 points that state P's position at $t = 1, t = 2, \ldots, t = 10$. Hint: Maple-commands `seq` and `pointplot`.

b) Find using Maple an antiderivative to the Jacobi function corresponding to $\mathbf{r}$. Explain that the function $f(u) = \operatorname{arcosh}(u^2)$, $u > 1$ measures the length of the part of $K_{\mathbf{r}}$ that P has moved in time from $t = 1$ to $t = u$.

c) Explain that the function $g(v) = \sqrt{\cosh(v)}$, $v > 0$ measures the time that has elapsed from $t = 1$ until P has traversed the curve length v.

d) Make a plot of $K_{\mathbf{r}}$ together with 10 positions of P that divides $K_{\mathbf{r}}$ in 9 pieces of equal length.

For a number $a \in \,]1, 5[$ we consider in the (x, z)-plane in space the profile curve $K_{\mathbf{s}}$ with the parametric representation

$$\mathbf{s}(u) = \left(u, 0, \operatorname{arcosh}(u) \right), \quad u \in [a, 5] .$$

A function f is given by

$$f(x, y, z) = \frac{1}{z \cdot (x^2 + y^2)} .$$

e) Determine a parametric representation for the surface of revolution that appears when $K_{\mathbf{s}}$ is rotated one turn about the z-axis, and determine for $a = 2$ the surface integral $\int_{K_{\mathbf{s}}} f \, \mathrm{d}\mu$ as a decimal number with 3 decimals.

f) Explain that the surface integral mentioned in the previous question is not defined for $a = 1$. Investigate the surface integral when a tends towards 1.

Hint: The Area functions arcosh and arsinh is written in Maple as arcus functions, i.e. arccosh and arcsinh (even though they have nothing to do with arc lengths...).

The assignment is finished