TECHNICAL UNIVERSITY OF DENMARK

Written 2-hours exam in the spring curriculum May 14, 2018.

Course Name: Advanced Engineering Mathematics 1. **Course no.** 01006

Allowed helping aids: All helping aids allowed by DTU can be brought and used.

Weighting: The four exercises are equally weighted.

All answers must be supported by valid arguments, and intermediate calculations should be included to an appropriate extend. It is not allowed to communicate with others during the exam, neither directly nor electronically.

PROBLEM 1

A real function f of two real variables is given by

$$f(x,y) = 4y\left(x^2+\frac{y^2}{3}-1\right) .$$

1. Determine all local extrema for f and state the points in which they are attained.

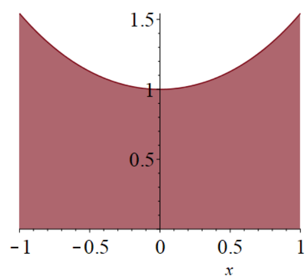
An ellipse E in the (x,y)-plane is given by the equation

$$x^2+\frac{y^2}{3}-1=0 .$$

2. Determine E's centre and semiaxes.
3. Determine the global maximum and the global minimum of f on the closed set of points that is bounded by E.

Problem 2

In the (x,y)-plane we consider the set of points $B=\{(x,y)\,|\,-1\leq x\leq 1 \text{ and } 0\leq y\leq \cosh(x)\}$.



1. Determine a parametric representation for B.

A height function h defined on the (x,y)-plane is given by

$$z=h(x,y)=2-x .$$

Let F denote the part of the graph for h that is vertically above B.

2. Find a parametric representation for F, and determine the Jacobi function corresponding to the parametric representation.
3. Determine the surface integral $\displaystyle\int_F \frac{z-1}{\sqrt{2}}\,\mathrm{d}\mu$.

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PROBLEM 3

It is given that the system of first-order linear differential equations

$$\begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} 4x(t) - 10y(t) \\ 2x(t) - 5y(t) \end{bmatrix}.$$

has the complete solution

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} 5c_1 + 2c_2\,\mathrm{e}^{-t} \\ 2c_1 + c_2\,\mathrm{e}^{-t} \end{bmatrix},\ c_1, c_2 \in \mathbb{R}.$$

In the (x,y)-plane we consider the vector field $\mathbf{V}(x,y) = \begin{bmatrix} 4x - 10y \\ 2x - 5y \end{bmatrix}$.

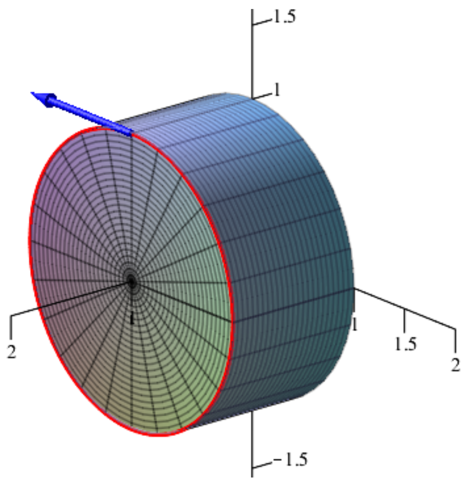
1. Determine the flow curve $\mathbf{r}(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$ for $\mathbf{V}$ that fulfills $\mathbf{r}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

A curve L is given as the straight line segment from the point $(1,1)$ to the point $(2,2)$.

2. Determine a parametric representation for L.
3. Now let L at time $t=0$ start to flow with $\mathbf{V}$ seen as a velocity vector field. Determine a parametric representation for the curve that L has been deformed to, at time $t=1$.

PROBLEM 4

In the (x,y,z)-space the vector field $\mathbf{V}(x,y,z) = (3x - y^2,\, 4yz\,,\, y - 2z^2)$ is given. Moreover, we consider a solid cylinder of revolution Ω with radius 1 that has the x-axis as the axis of symmetry and is bounded by the planes $x=0$ and $x=1$, see the figure.



1. Determine the flux of $\mathbf{V}$ out through the surface $\partial\Omega$ of Ω.

Consider the circular disc C lying in the plane $x=1$ that ends the cylinder in the direction of the x-axis, and its boundary curve ∂C that is shown in red in the figure.

2. Find a parametric representation $\mathbf{r}(u,v)$ for C, that fulfills the right-hand rule in relation to the orientation of ∂C that is shown in the figure by the blue vector.
3. Determine the circulation $\oint_{\partial C} \mathbf{V} \cdot \mathbf{e}_{\partial C}\,\mathrm{d}\mu$.

THE END