THE TECHNICAL UNIVERSITY OF DENMARK

Written 2-hour test in the Spring syllabus, May 11 2022.

Course title: Advanced Engineering Mathematics 1. **Course no.:** 01006

Permitted aids: You may bring and use all aids permitted by DTU.

Weight: The four problems weigh equally.

All answers must be well-reasoned, and relevant calculations must be shown to an appropriate extent. No communication with others during the test is allowed, either directly or electronically.

PROBLEM 1

For a function $f:\mathbb{R}^2\to\mathbb{R}$ the two 1st-order partial derivatives are given:

$$f'_x(x,y)=6x-6y \quad \text{and} \quad f'_y(x,y)=6y^2-6x\,.$$

1. Determine all stationary points for f.
2. Determine the 2nd-order partial derivatives of f and set up the Hessian matrix of f. Determine the points in the (x,y) plane at which f has local maxima or local minima.
3. It is now given that $f(0,0)=1$. Determine the approximating 2nd-degree polynomial for f with expansion point $(0,0)$.

PROBLEM 2

A rectangle M_1 is located in the (x,y) plane in (x,y,z) space and is given by:

$$M_1=\{\,(x,y)\,|\,0\le x\le 2\,,\ 0\le y\le 1\,\}\,.$$

We consider a function $h:\mathbb{R}^2\to\mathbb{R}$ given by the expression $h(x,y)=2x-y+1$. Let G_1 denote the part of the graph of h that is located vertically above M_1.

1. Compute the area of G_1.

The straight line segment between the points $(0,1)$ and $(2,0)$ divides M_1 in two parts. Let M_2 denote the lower part, and let G_2 denote the part of the graph of h that is located vertically above M_2.

2. Provide a parametric representation of G_2, and determine its Jacobian function.
3. Compute the surface integral of f over G_2 when f is given by the expression

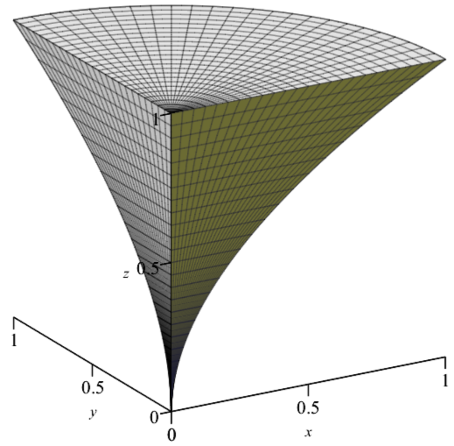
$$f(x,y,z)=x+y+z-1\,,\ (x,y,z)\in\mathbb{R}^3\,.$$

The problem sheet continues ⟶

PROBLEM 3

A solid body L in (x,y,z) space is given by the parametric representation

$$\mathbf{r}(u,v,w)=\left(v\cdot u^2\cdot\cos(w),\, v\cdot u^2\cdot\sin(w),\, u\right),\ u\in[0,1],\, v\in[0,1],\ w\in\left[0,\frac{\pi}{2}\right].$$



1. Determine the Jacobian function of $\mathbf{r}$.
2. We consider the vector field $\mathbf{V}(x,y,z)=(x+e^{y\cdot z},\, 2y-e^{x\cdot z},\, 3z+e^{x\cdot y})$. Compute the flux of $\mathbf{V}$ out through the surface ∂L of L.
3. L is formed by rotating a profile area F in the (x,z) plane (marked yellow on the figure) a quarter of a full rotation about the z axis counterclockwise as seen from the positive end of the z axis. Provide a parametric representation of F.

PROBLEM 4

Let a be a positive real number. A flow curve K of a smooth vector field $\mathbf{V}$ in (x,y,z) space (whose expression is not given) is given by the parametric representation

$$\mathbf{r}(t)=\begin{bmatrix} e^{-t} \\ \cos(t)-\sin(t) \\ \cos(t)+\sin(t)\end{bmatrix},\ t\in[0,a].$$

1. The Jacobian function of $\mathbf{r}$ can be written in the form
$$\text{Jacobian}(t)=\sqrt{e^{-m\cdot t}+n},$$
where m and n are real numbers. Determine m and n.
2. Justify that the tangential line integral of $\mathbf{V}$ along K can be computed as
$$\int_K \mathbf{V}\cdot\mathbf{e}\,d\mu=\int_0^a \mathbf{r}'(t)\cdot\mathbf{r}'(t)\,dt=\int_0^a \left(\text{Jacobian}(t)\right)^2 dt.$$
3. Let $a=5$ and compute $\int_K \mathbf{V}\cdot\mathbf{e}\,d\mu$.

End of the problem sheet.