TECHNICAL UNIVERSITY OF DENMARK

Written 2-hours exam in the spring curriculum May 17, 2021: Essay Assignment Part.

Course Name: Advanced Engineering Mathematics 1.

Course no. 01006

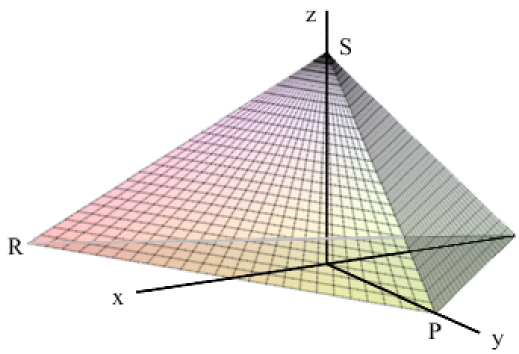
Allowed helping aids: All helping aids allowed by DTU can be used. For other rules, please refer to the document 'Syllabus & Rules' on the course homepage

Essay Assignment

In (x,y,z)-space we are given the vector fields

$$\mathbf{V}(x,y,z) = \left(z^2+y,\, 3y-1,\, x^2+xz\right) \text{ and } \mathbf{U}(x,y,z) = \mathbf{Curl}(\mathbf{V})(x,y,z)$$

and a solid tetrahedron T spanned by the points $P=(0,1,0)$, $Q=(-1,0,0)$, $R=(1,-1,0)$ and $S=(0,0,1)$.



1. Find a parametric representation for the straight line from P to Q, and determine the tangential curve integral of $\mathbf{V}$ along the line segment. Also, determine the tangential curve integral of $\mathbf{V}$ from P to Q along the broken straight line that goes through R. Is $\mathbf{V}$ a gradient vector field?

The filled triangle F_1 between the points P, Q and S can be parametrized by

$$\mathbf{s}(u,v) = \left(-uv,\, v(1-u),\, 1-v\right),\ u \in [0,1]\ ,\ v \in [0,1]\ .$$

2. Compute the normal vector $\mathbf{N}(u,v) = \mathbf{s}'_u(u,v) \times \mathbf{s}'_v(u,v)$ and determine the flux of $\mathbf{U}$ through F_1. Choose an orientation of the boundary ∂F_1 that fulfills the right-hand rule in relation to $\mathbf{N}$, and determine the circulation $\mathbf{V}$ along ∂F_1.

Let F_2 denote the filled triangle between the points P, Q and R which is thought to be oriented with downward unit normal vector.

3. Determine the flux of $\mathbf{V}$ through F_2.

4. Determine a parametric representation for the given tetrahedron T, and determine the flux of $\mathbf{V}$ out through the surface of the tetrahedron ∂T.

5. Assume that T at time 0 begins to flow with $\mathbf{U}$ perceived as a velocity vector field, we follow especially the triangle F_1. Determine the area of the surface to which F_1 has been deformed at time 1.

THE END