TECHNICAL UNIVERSITY OF DENMARK
**Written 2-hours exam** in the spring curriculum May 14, 2019.
**Course Name:** Advanced Engineering Mathematics 1. **Course no.** 01006
**Allowed helping aids:** All helping aids allowed by DTU can be brought and used.
**Weighting:** The four exercises are equally weighted.
All answers must be supported by valid arguments, and intermediate calculations should be included to an appropriate extend. It is not allowed to communicate with others during the exam, neither directly nor electronically.

## PROBLEM 1

A real function $f$ of two real variables are given by

$$f(x,y) = \frac{y}{\mathrm{e}^x}\,.$$

1. Determine the level curves for $f$ corresponding to the levels 0 and 1.
2. Determine the directional derivative of $f$ in the point $(0,1)$ in the direction towards the point $(1,2)$, and find the largest value that the directional derivative of $f$ in $(0,1)$ can attain.

A curve $K$ in the $(x,y)$-plane is given by the parametric representation

$$\mathbf{r}(u) = \begin{bmatrix} u \\ 1+u-u^2 \end{bmatrix},\ u \in \mathbb{R}\,.$$

3. Determine the points on $K$ where the gradient of $f$ and the tangent vector $\mathbf{r}'(u)$ are orthogonal.

## PROBLEM 2

It is given that the matrix

$$\mathbf{A} = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

has the eigenvalues 1 and 3 with corresponding eigenspaces $E_1 = \operatorname{span}\{(1,1)\}$ and $E_3 = \operatorname{span}\{(-1,1)\}$.
A quadratic polynomial $f : \mathbb{R}^2 \to \mathbb{R}$ is given by

$$f(x,y) = 2x^2 - 2xy + 2y^2 - 4x + 2y + 2\,.$$

1. The expression for $f$ includes a quadratic form. State a positive orthogonal change of base matrix $\mathbf{Q}$ that reduces the quadratic form such that it in the new $(x_1,y_1)$-coordinates does not contain ”mixed terms”, i.e. product terms of the type $x_1 \cdot y_1$.
2. If one uses a change of base as described in the previous question, $f$ can be written in the reduced form
$$f_1(x_1,y_1) = a(x_1-c)^2 + b(y_1-d)^2\,.$$
Determine numbers $a,b,c$ and $d$ that fulfill this, and let $(x_0,y_0)$ denote the $(x,y)$-coordinates for the point $(c,d)$. Explain that $f$ has a local minimum in $(x_0,y_0)$.

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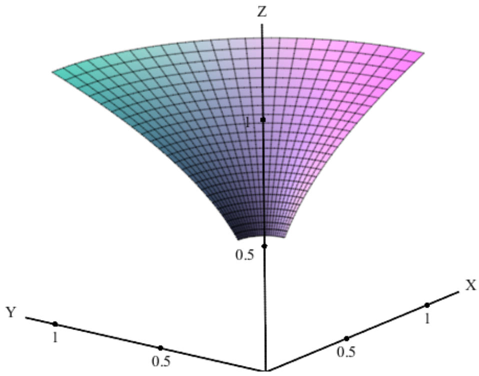
## PROBLEM 3

Let $a$ be a positive real number. A surface $F$ in the $(x,y,z)$-space is given by the parametric representation $\mathbf{r}(u,v) = \left(u^3\cos(v),\, u^3\sin(v),\, u\right)$, $u\in[1/2,\,1]$, $v\in[0,\,a]$, shown in the figure for $a=\frac{\pi}{2}$.



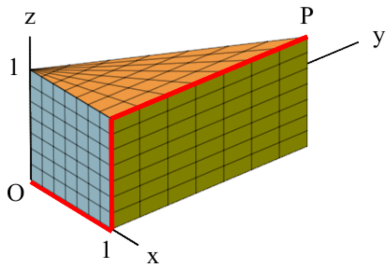

1. Explain that $F$ is a surface of revolution and draw its profile curve in the $(x,z)$-plane that is used in the rotation.
2. Determine the Jacobi function corresponding to $\mathbf{r}$.
3. Consider a mass density function in the $(x,y,z)$-space that in every point on $F$ has the same value as the Jacobi function corresponding to $\mathbf{r}$. What shall be the value of $a$ so that $F$ attains the mass 1?

## PROBLEM 4

A triangle $T$ in the $(x,y)$-plane has the vertices $(0,0),(1,0)$ and $(1,2)$.

1. State a parametric representation for $T$.

By $\Omega$ we denote the point set in the $(x,y,z)$-space that lies vertically between $T$ and the plane with the equation $z=1$.




2. State a parametric representaion for $\Omega$ and determine the Jacobi function corresponding to the parametric representation.

A function $f:=\mathbb{R}^3\to\mathbb{R}$ is given by $f(x,y,z)=x^2yz$. We consider the vector field $\mathbf{V}(x,y,z)=\nabla f(x,y,z)$.

3. Determine the flux
$$\int_{\partial\Omega}\mathbf{V}\cdot\mathbf{n}\,\mathrm{d}\mu\,.$$
4. A curve $K$ that runs along three edges of $\Omega$ from the origin to the point $P(1,2,1)$, is shown in red on the Figure. Determine the tangential curve integral
$$\int_K\mathbf{V}\cdot\mathbf{e}\,\mathrm{d}\mu\,.$$

THE END