TECHNICAL UNIVERSITY OF DENMARK
Written in exam in spring curriculum May 13, 2017.
Course Name: Advanced Engineering Mathematics 1. **Course no.** 01006
Allowed helping aids: All helping aids allowed by DTU.
Weighting: The four exercises are equally weighted.
All answers should be supported by valid arguments, and intermediate calculations should be included to an appropriate extend. It is not allowed to communicate with others during the exam, neither directly nor electronically.

PROBLEM 1

A function f of two real variables is for $(x,y) \neq (0,0)$ given by

$$f(x,y) = \frac{y}{x^2+y^2}\,.$$

1. We consider three points in the (x,y)-plane: $A=(0,1)$, $B=(0,-1)$ and $C=\left(\frac{1}{2},\frac{1}{2}\right)$. Exactly two of these lie on the same level curve for f. Which two?

The gradient of f is given by $\nabla f(x,y) = \left(\dfrac{-2xy}{(x^2+y^2)^2}, \dfrac{x^2-y^2}{(x^2+y^2)^2}\right)$.

2. Explain that f has no stationary points.

Consider the closed and bounded set of points (a circular ring) $M = \left\{(x,y)\,|\,1 \leq x^2+y^2 \leq 4\right\}$.

3. Determine the global minimum and the global maximum of f on M, and state the points where the global maximum and the global minimum are attained.

PROBLEM 2

The approximating second-degree polynomial about the point $(0,0)$ of the function $f(x,y)$ is given by

$$P_2(x,y) = 2 + \frac{1}{2}x^2 + y^2$$

while the approximating second-degree polynomial about the point $\left(\frac{\sqrt{6}}{3},0\right)$ is given by

$$Q_2(x,y) = \frac{4}{3}\sqrt{\mathrm{e}} - \sqrt{\mathrm{e}}\left(x-\frac{\sqrt{6}}{3}\right)^2 + \frac{1}{3}\sqrt{\mathrm{e}}\,y^2\,.$$

1. Determine the functional values

$$f(0,0),\ f'_x(0,0),\ f'_y(0,0),\ f''_{xx}(0,0),\ f''_{yy}(0,0) \text{ and } f''_{xy}(0,0)\,.$$

2. Explain that $(0,0)$ is a stationary point for f, and investigate whether $f(0,0)$ is a proper local minimum, a proper local maximum or neither of these.

3. Explain that $\left(\frac{\sqrt{6}}{3},0\right)$ is also a stationary point for f, and investigate whether $f\left(\frac{\sqrt{6}}{3},0\right)$ is a proper local minimum, a proper local maximum or neither of these.

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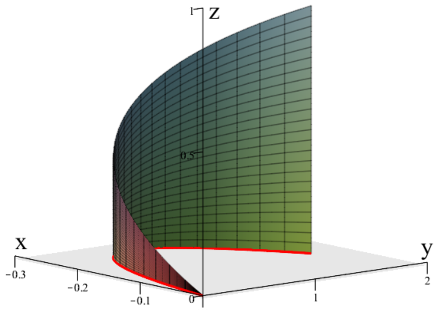
PROBLEM 3

A cylindrical surface F in the (x,y,z)-space is given by the parametric representation $\mathbf{r}(u,v) = \left(u^2 - u\,,\, u^2 + u\,,\, vu\right)$ where $u \in \left[0, \frac{\sqrt{3}}{2}\right]$ and $v \in [0,1]$.



1. Determine the Jacobi function for $\mathbf{r}$, and use this to determine the area of F.

Let L denote the directrix in the (x,y)-plane (shown in red in the figure) corresponding to F.

2. State a parametric representation of L, and determine the Jacobi function for L.
3. Determine the curve integral $\int_L \frac{1}{2}(y-x)\,\mathrm{d}\mu$.

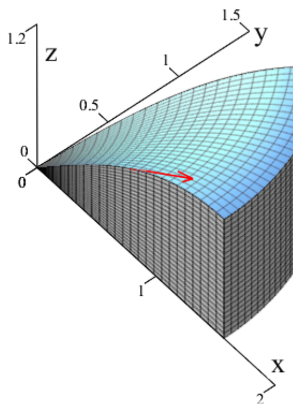
PROBLEM 4

A vector field in the (x,y,z)-space is given by $\mathbf{V}(x,y,z) = (x^2, -2yx\,, z)$. In the (x,z)-plane we consider a profile region A given by the parametric representation

$$\mathbf{s}(u,v) = (u, 0, v \cdot \sin(u))\,,$$

where $u \in \left[0, \frac{1}{2}\pi\right]$ and $v \in [0,1]$.

A solid of revolution Ω appears by rotating A through the angle $\frac{\pi}{4}$ about the z-axis counter-clockwise as seen from the positive end of the z-axis.



1. Give a parametric representation for Ω.
2. Determine the flux of $\mathbf{V}$ out through the surface of Ω.
3. Let G denote the part of the surface of Ω, that bounds Ω above (blue in the figure). Determine the circulation of $\mathbf{V}$ along the boundary of G when the boundary curve is oriented as indicated by the arrow in the figure.

THE END