Danmarks Tekniske Universitet
Written 2-hours test spring curriculum, May 17, 2016.
Course name: Matematik 1. **Course no.** 01005
Allowed helping aids: All helping aids allowed by DTU.
Weighting: The four problems are equally weighted.
All answers should be supported by valid arguments, and intermediate calculations should be included to an appropriate extend. It is not allowed to communicate with others during the exam, neither directly nor electronically.

PROBLEM 1

A function $f : \mathbb{R} \to \mathbb{R}$ is given by

$$f(x) = \sqrt{2x-1}.$$

1. Determine the domain $\mathrm{Dm}(f)$ of f.
2. Find the approximating polynomial $P_3(x)$ of third degree for f based at $x_0 = 1$.
3. Explain why the remainder function $R_3(x)$ corresponding to $P_3(x)$ is given by

$$R_3(x) = -\frac{5}{8} \cdot \frac{1}{(2\xi-1)^{7/2}} \cdot (x-1)^4$$

for a ξ between 1 and x. Show by estimating the remainder function that the absolut value of the difference between $P_3\left(\frac{3}{2}\right)$ and $f\left(\frac{3}{2}\right)$ is less than or equal to $\frac{5}{2^7}$.

PROBLEM 2

A symmetric matrix is given:

$$\mathbf{A} = \begin{bmatrix} \frac{288}{25} & \frac{84}{25} \\ \frac{84}{25} & \frac{337}{25} \end{bmatrix}.$$

1. Find in $\mathbb{R}^{2\times 2}$ a positive orthogonal matrix $\mathbf{Q}$ and a diagonal matrix Λ such that

$$\Lambda = \mathbf{Q}^{-1}\mathbf{A}\mathbf{Q}.$$

An ellipse $\mathcal{E}$ is in the standard orthogonal (x, y)-coordinate system in the plane is given by the matrix equation

$$\begin{bmatrix} x & y \end{bmatrix} \mathbf{A} \begin{bmatrix} x \\ y \end{bmatrix} = 144.$$

2. Determine the semiaxes of $\mathcal{E}$.

Continues on next page $\longrightarrow$

PROBLEM 3

For a smooth function $f : \mathbb{R}^2 \to \mathbb{R}$ with $f(0,0) = 0$, a vector field $\mathbf{V}$ in the (x,y)-plane is given by

$$\mathbf{V}(x,y) = \nabla f(x,y) = (x - y^2 + 1, -2xy).$$

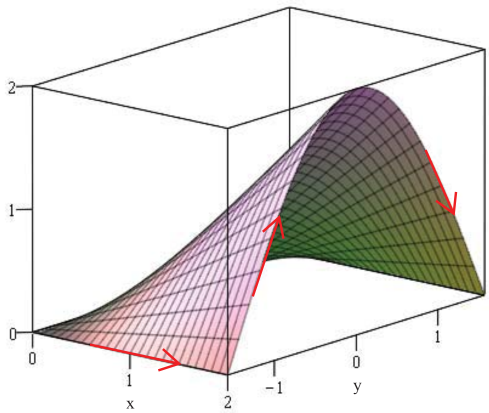
1. Find all the stationary points for f.
2. Find the Hessian matrix of f, and explain why f has exactly one strict local minimum point and no strict local maximum point.
3. Find the line integral of $\mathbf{V}$ along a curve $\mathcal{K}$ of your own choice from the origon to an arbitrary point (x,y). Hint: You may use the formula

   $$(x,y) \cdot \int_0^1 \mathbf{V}(ux, uy)\mathrm{d}u.$$

   Or, you can integrate along the broken line in the (x,y)-plane that connects $(0,0)$ and $(x,0)$, and $(x,0)$ and (x,y).
4. Find the value of f at the strict local minimum point you found in Question 2.

PROBLEM 4

In the (x,y)-plane in the (x,y,z)-space, a set $A = \{(x,y) \mid 0 \le x \le 2 \text{ and } -\frac{\pi}{2} \le y \le \frac{\pi}{2}\}$ and a function $h(x,y) = x\cos(y)$ are given. Let $\mathcal{F}$ denote the part of the graph of h that is vertically above A, see the figure.



1. Find a parametrization $\mathbf{r}(u,v)$ for $\mathcal{F}$ and find the normal vector $\mathbf{N}(u,v) = \mathbf{r}'_u(u,v) \times \mathbf{r}'_v(u,v)$ corresponding to $\mathbf{r}(u,v)$.

Suppose $\mathbf{V}$ is a vector field in the (x,y,z)-space and that $\mathrm{Div}(\mathbf{V})(x,y,z) = x + y + z$ and $\mathbf{Rot}(\mathbf{V})(x,y,z) = (3z, 3x, 3y)$.

2. Find the line integral (circulation) of $\mathbf{V}$ along the closed boundary curve $\partial\mathcal{F}$ of $\mathcal{F}$ where the orientation of $\partial\mathcal{F}$ is as shown of the figure above.
3. Let Ω denote the 3-dimensional solid region that lies vertically between A and $\mathcal{F}$. Find the flux of $\mathbf{V}$ out of the closed surface $\partial\Omega$ of Ω.

The end