DANMARKS TEKNISKE UNIVERSITET

Written 2-hours test spring curriculum, May 18, 2015.

Kursus Navn: Advanced Engineering Mathematics 1. **Course no.** 01005

Allowed helping aids: All helping aids allowed by DTU.

Weighting: The four problems are equally weighted.

All answers should be supported by valid arguments, and intermediate calculations should be included to an appropriate extend. It is not allowed to communicate with others during the exam, neither directly nor electronically.

PROBLEM 1

A smooth function $f : \mathbb{R}^2 \to \mathbb{R}$ is given by

$$f(x,y) = x^2 y + \frac{1}{3} y^3 - y\,.$$

1. Find all stationary points for f.
2. Investigate every one of the points $(1,0)$, $(0,0)$, $(0,1)$ whether f has a proper local maximum, a proper local minimum or neither of these in the point.
3. Determine all directions from the point $(0,0)$ where the directional derivative of f in $(0,0)$ assumes the value 0.

PROBLEM 2

It is given that the matrix

$$\mathbf{A} = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{3}{2} \end{bmatrix}$$

has the eigenvector $\mathbf{v}_1 = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ corresponding to the eigenvalue 2 and the eigenvector $\mathbf{v}_2 = \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ corresponding to the eigenvalue 0.

1. Explain that the set of vectors $(\mathbf{v}_1, \mathbf{v}_2)$ is an orthonormal basis for $\mathbb{R}^2$.

A parabola is in an ordinary orthogonal (x,y)-coordinate system in the plane given by the equation

$$\frac{1}{2}x^2 + \sqrt{3}\,xy + \frac{3}{2}y^2 + \frac{\sqrt{3}}{2}x - \frac{1}{2}y = -2\,.$$

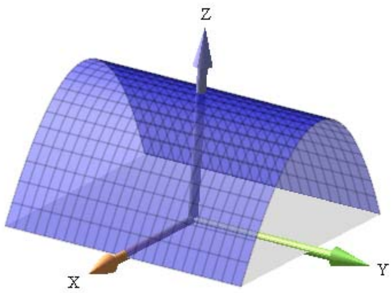
2. Determine the vertex and the axis of symmetry of the parabola.

PROBLEM 3

The function $h : \mathbb{R}^2 \to \mathbb{R}$ is given by $h(x,y) = 1 - x^2$. We consider the graph surface F given by

$$F = \{\,(x,y,z) \mid -1 \le x \le 1,\ -1 \le y \le 1,\ z = h(x,y)\,\}\,.$$

Continues on next page ⟶



1. Determine a parametric representation $\mathbf{r}(u,v)$ for F which fulfills that the normal vector $\mathbf{N}(u,v) = \mathbf{r}'_u(u,v) \times \mathbf{r}'_v(u,v)$ has positive z-coordinate.

A vector field $\mathbf{V}$ is given by $\mathbf{V}(x,y,z) = (x^2,\, z-2xy,\, 4z)$.

2. Determine the flux $\displaystyle\int_{F_\mathbf{r}} \mathbf{V}\cdot\mathbf{n}_{F_\mathbf{r}}\,\mathrm{d}\mu$.

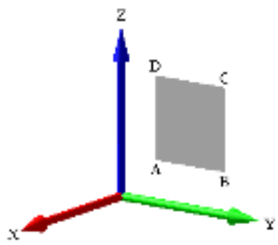
Let Ω denote the 3-dimensional solid region that lies vertically between the (x,y)-plane and F.

3. Determine the volume of Ω.

4. Determine the flux of $\mathbf{V}$ out of the surface $\partial\Omega$ of Ω.

PROBLEM 4

In (x,y,z)-space the points $A(0,1,1)$, $B(0,3,1)$, $C(0,3,3)$ and $D(0,1,3)$ are given.



The plane square spanned by the four points is denoted K. Furthermore we consider the vector field $\mathbf{U}$ given by $\mathbf{U}(x,y,z) = (2xy, -z^2, x^2)$ and the vector field $\mathbf{V}$ that is the gradient of the function $f(x,y,z) = x^2y - \dfrac{z^3}{3}$.

1. Determine the flux of $\mathbf{curl}(\mathbf{U})$ through K, when K is parametrized by

$$\mathbf{r}(u,v) = (0,u,v) \text{ med } u \in [1,3] \text{ og } v \in [1,3]\,.$$

2. Determine the tangential curve integral of both $\mathbf{U}$ and $\mathbf{V}$ along the straight line segment from A to D.

3. Determine the circulation of both $\mathbf{U}$ and $\mathbf{V}$ along the boundary K the direction of which is given by the order of points $ABCDA$.

The end