DANMARKS TEKNISKE UNIVERSITET

Written 2-hours test spring curriculum, May 17, 2014.

Kursus Navn: Advanced Engineering Mathematics 1. **Course no.** 01005

Allowed helping aids: All helping aids allowed by DTU.

Weighting: The four problems are equally weighted.

All answers should be supported by valid arguments, and intermediate calculations should be included to an appropriate extend. It is not allowed to communicate with others during the exam, neither directly nor electronically.

PROBLEM 1

A function f is given by

$$f(x,y) = 2\cos(x) - \sin(2x)\,, x \in \mathbb{R}\,.$$

1. Determine by means of elementary theorems about differentiation the derivatives

$$f'(x), f''(x) \text{ and } f'''(x)\,.$$

Let $P_2(x)$ denote the approximating second-degree polynomial for f with the development point $x_0 = 0$, and let R_2 denote the corresponding remainder function given by

$$R_2(x) = f(x) - P_2(x) = \frac{f'''(\xi)}{3!}x^3 \text{ for a } \xi \text{ between } 0 \text{ and } x\,.$$

2. Use the results from Question 1 to form P_2.

3. Show by evaluation of R_2 that the error committed by using $P_2(\frac{1}{10})$ instead of $f(\frac{1}{10})$ is less than of equal to $\frac{1}{600}$.

PROBLEM 2

A function $f : \mathbb{R}^2 \to \mathbb{R}$ is given by

$$f(x,y) = \frac{1}{2}y^2 - y\sin(x)\,.$$

Furthermore the open set of points M is considered

$$M = \{(x,y) \in \mathbb{R}^2 \mid -3 < x < 3\}$$

1. Determine the partial derivatives of first and second order for f.

2. Given that f has three stationary points belonging to M. Determine these three stationary points.

3. Determine all points in M in which f has a local minimum, and all points in M in which f has a local maximum.

PROBLEM 3

In $(x,y,z)-$ space a space curve $\mathcal{K}_\mathbf{r}$ is given by the parametric representation

$$\mathbf{r}(t) = (e^t - e^{-t}, e^t + e^{-t}, 1-2t), t \in [1,1].$$

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1. Show that $|\mathbf{r}'(t)| = \sqrt{2}(\mathrm{e}^t + \mathrm{e}^{-t})$, and determine the length of $\mathcal{K}_\mathbf{r}$.

A first order vector field $\mathbf{V}$ is given by $\mathbf{V}(x,y,z) = (y, x, -2)$.

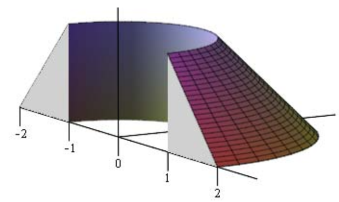
2. Determine (with all intermediate results) the tangential curve integral of $\mathbf{V}$ along $\mathcal{K}_\mathbf{r}$.

3. Show that $\mathcal{K}_\mathbf{r}$ is the flow curve for $\mathbf{V}$ that begins in the points $(0,2,1)$ at time $t = 0$.

PROBLEM 4

A solid region Ω in (x,y,z)-space is given by the parametric representation

$$\mathbf{r}(u,v,w) = (u\cos(w), u\sin(w), v(2-u)) \text{ where } u \in [1,2], v \in [0,1], w \in [0,\pi].$$



1. Two functions $f : \mathbb{R}^3 \to \mathbb{R}$ and $g : \mathbb{R}^3 \to \mathbb{R}$ are given by $f(x,y,z) = 1$ and $g(x,y,z) = \frac{y}{2}$, respectively. Determine (with all intermediate results) the space integrals

$$\int_\Omega f \,\mathrm{d}\mu \text{ and } \int_\Omega g \,\mathrm{d}\mu.$$

Consider the vector field $\mathbf{V}$ given by $\mathbf{V}(x,y,z) = (\frac{1}{2}z^2, \frac{1}{4}y^2, -2y)$.

2. Determine the flux of $\mathbf{V}$ out through surface $\partial\Omega$ of Ω.

3. State a vector field $\mathbf{U}$ with constant divergence that fulfills

$$\int_z \mathrm{U} \cdot \mathbf{n}_{\partial\Omega} \mathrm{d}\mu = 2\pi.$$

Ω appears through a rotation of plane triangular region T lying in the $(x,z)-$plane the angle π around the $z-$axis in the positive direction.

4. State a parametric representation for T, and determine the circulation of $\mathbf{V}$ along the boundary curve $\partial T$of T with the direction shown in the figure.

The end