TECHNICAL UNIVERSITY OF DENMARK

Written 2-hours exam in the fall curriculum December 9, 2018.

Course Name: Advanced Engineering Mathematics 1. **Course no.** 01006

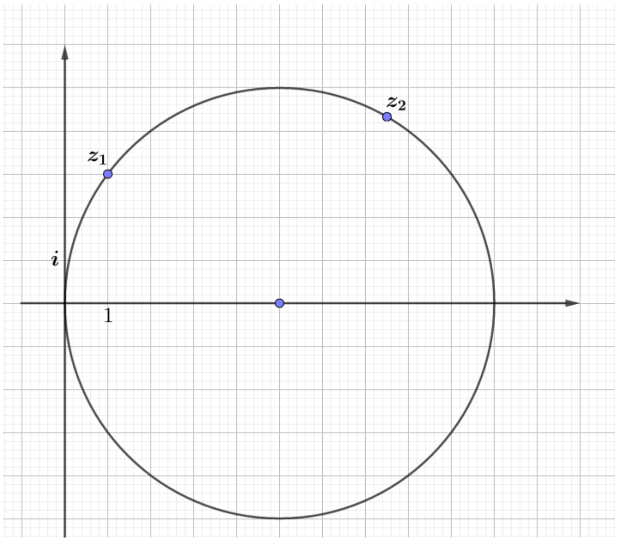
Allowed helping aids: All helping aids allowed by DTU can be brought and used.

Weighting: The four exercises are equally weighted.

All answers must be argued and intermediate calculations included to an appropriate extend.

PROBLEM 1

A circle C in the complex number plane is given by the equation $|z-5|=5$.



1. Show that the number $z_1 = 1+3i$ lies on C, and that $\operatorname{Re}\left(\frac{1}{z_1}\right) = \frac{1}{10}$.

2. Show that the number $z_2 = 5\sqrt{3}\,e^{i\frac{\pi}{6}}$ fulfills $\operatorname{Re}\left(\frac{1}{z_2}\right) = \frac{1}{10}$.

3. Show that every number z that fulfills $\operatorname{Re}\left(\frac{1}{z}\right) = \frac{1}{10}$ lies on C.

PROBLEM 2

Let $\mathbf{A}$ denote the coefficent matrix and $\mathbf{T}$ the augmented matrix for an inhomogeneous system of linear equations that has the totally reduced form

$$\operatorname{trap}(\mathbf{T}) = \begin{bmatrix} 1 & 0 & -4 & 0 & -2 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 3 & -5 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

1. State the complete solution to the system of equations in standard parameter form.

A linear map $f:\mathbb{R}^5 \to \mathbb{R}^4$ is given by its mapping matrix $\mathbf{A}$ (the coefficient matrix from question 1) with respect to the standard bases in $\mathbb{R}^5$ and $\mathbb{R}^4$.

2. Determine the dimension of $\ker(f)$ and the dimension of the range $f(\mathbb{R}^5)$.

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PROBLEM 3

In the vector space $\mathbb{R}^3$ equipped with the usual dot product, we are given the set of vectors

$$v = (\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3) = ((1,1,1),(1,0,-1),(-1,1,0))\,.$$

1. Explain that v is a basis for $\mathbb{R}^3$.

A linear map $f:\mathbb{R}^3 \to \mathbb{R}^3$ is given by the fact that for every vector $\mathbf{u} \in \operatorname{span}\{\mathbf{v}_1\}$ it applies that

$$f(\mathbf{u}) = 5\mathbf{u}\,,$$

and for every $\mathbf{u} \in \operatorname{span}\{(\mathbf{v}_2, \mathbf{v}_3)\}$ it applies that

$$f(\mathbf{u}) = -4\mathbf{u}\,.$$

2. Determine the mapping matrices ${}_v\mathbf{F}_v$ and ${}_e\mathbf{F}_e$ for f with respect to the base v, and the standard base e for $\mathbb{R}^3$, respectively.
3. Find an orthonormal basis for $\mathbb{R}^3$ consisting of eigenvectors for f, that fulfills that one of the basis vectors is aligned with $\mathbf{v}_3$.

PROBLEM 4

Let a be an arbitrary real number. A system of linear first-order differential equations is given by

$$\begin{bmatrix} x_1{}'(t) \\ x_2{}'(t) \end{bmatrix} = \begin{bmatrix} a & 3-a \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix},\ t \in \mathbb{R}\,.$$

1. Explain that when $a = 3$, the system of equations cannot be solved by the method of diagonalization.
2. For another value of a

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \mathrm{e}^{(4+i)t} \begin{bmatrix} 1+i \\ 1 \end{bmatrix} = \begin{bmatrix} (1+i)\,\mathrm{e}^{(4+i)t} \\ \mathrm{e}^{(4+i)t} \end{bmatrix}$$

   is a particular solution to the system of differential equations. State for this value of a the complete complex solution to the system of differential equations.

3. Determine the value of a that was used in question 2.

THE END