TECHNICAL UNIVERSITY OF DENMARK
Written examination December 9, 2013.
Course Name: Mathematics 1.
Course no 01005
Allowed helping aids: All helping aids allowed by DTU can be brought and used.
Weight: The four problems will have equal weight.
All answers should be supported by valid arguments, and intermediate calculations should be included to an appropriate extend. It is not allowed to communicate with others during the exam, neither directly nor electronically.

Problem 1

About an inhomogeneous system of linear equation it is stated that its augmented matrix $\mathbf{T}$ by complete reduction has the form

$$\text{trap}(\mathbf{T}) = \begin{bmatrix} 1 & 0 & 0 & 2 & -2 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 4 \end{bmatrix}.$$

1. State the number equations and unknowns in the system of equations. Furthermore determine the rank of the coefficient matrix and the augmented matrix belonging to the system of equations.
2. Determine which of the following set of numbers belongs to the solution set of the system of equations:

$$\mathbf{x}_1 = (1,0,4,1)\,,\ \mathbf{x}_2 = (1,-2,0,1)\,,\ \mathbf{x}_3 = (0,-1,4,-1)\,.$$

3. Write in standard parametric form the solution set for the homogeneous system of equations corresponding to the given system of equations.

Problem 2

In $\mathbb{R}^3$ the following three vectors are given

$$\mathbf{v}_1 = (1,0,1)\,,\ \mathbf{v}_2 = (0,1,0)\,,\ \mathbf{v}_3 = (1,0,-1)\,.$$

1. Show that the set of vectors $(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$ is a basis for $\mathbb{R}^3$.

About a real 3×3 matrix $\mathbf{A}$ it is stated that $\mathbf{A}\mathbf{v}_1 = \mathbf{v}_1$, $\mathbf{A}\mathbf{v}_2 = 2\mathbf{v}_2$ og $\mathbf{A}\mathbf{v}_3 = -\mathbf{v}_3$.

2. Show that $1, 2$ and -1 are eigenvalues for $\mathbf{A}$, and state their algebraic and geometric multiplicities.

Let $f : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear map that with respect to the standard basis for $\mathbb{R}^3$ has the mapping matrix $\mathbf{A}$.

3. Find the mapping matrix for f with respect to basis $(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.
4. Determine $\mathbf{A}$.
5. Given the vector $\mathbf{u} = (a,b,c) \in \mathbb{R}^3$, solve the equation $f(\mathbf{x}) = \mathbf{u}$.

The set of problems continues ⟶

Problem 3

1. Solve the homogeneous first-order linear differential equation $\frac{\mathrm{d}}{\mathrm{d}t}x(t) + t \cdot x(t) = 0$.

A linear map $f : \mathrm{C}^1(\mathbb{R}) \rightarrow \mathrm{C}^0(\mathbb{R})$ is given by

$$f(x(t)) = \frac{\mathrm{d}}{\mathrm{d}t}x(t) + t \cdot x(t).$$

2 State the kernel of f.

3 The function $q(t)$ is given by $q(t) = f(t-2)$. Determine the complete solution to the equation

$$f(x(t)) = q(t).$$

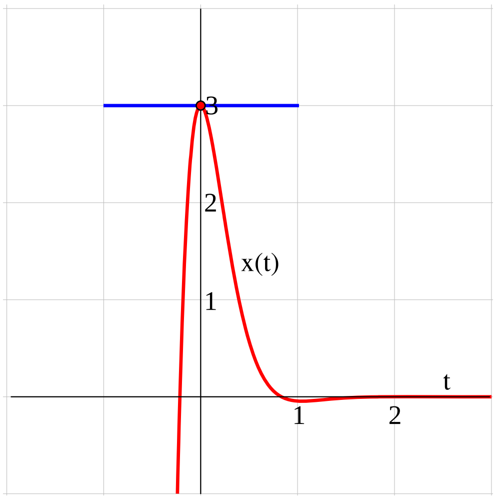
Problem 4

Let a be a real number. A linear homogeneous second-degree differential equation with constant coefficients has the characteristic equation

$$\lambda^2 + a \cdot \lambda + 25 = 0.$$

1. State the differential equation, when the unknown function is denoted $x(t)$.
2. Determine for $a = 0$ all solutions to the characteristic equation and use these solutions to find all solutions to the differential equation.
3. Determine the value of a a for which $x(t) = \mathrm{e}^{-4t} \cdot \cos(3t)$ is a solution to the differential equation.

   The Figure shows the graph for a particular solution to the differential equation with $a = 8$. the Figure also shows the tangent of the graph in the point of tangency $(0,3)$.



4. Determine the particular solution – shown in the Figure – to the differential equation.

`The set of problems is finished.`