THE TECHNICAL UNIVERSITY OF DENMARK
Written 2-hr test in the Spring curriculum, May 13 2024.
Course name: Advanced Engineering Mathematics 1. **Course no.** 01006
Permitted aids: You may bring and use all by DTU permitted aids.
Weights: The four problems weigh equally.
All answers must be well-reasoned, and in-between calculations must be shown to an appropriate extent.
You are not allowed to communicate with others during the test, not directly nor electronically.

PROBLEM 1

We are given the following information about a smooth function $f:\mathbb{R}^2 \to \mathbb{R}$, whose expression is not known:

$$f(0,1) = 1\,,\; f'_x(0,1) = 0\,,\; f'_y(0,1) = 0\,,\; f''_{xx}(0,1) = 1\,,\; f''_{yy}(0,1) = 0\,,\; f''_{xy}(0,1) = -1\,.$$

1. Create the approximating polynomial $P_2(x,y)$ of the second degree for f with the expansion point $(x_0,y_0) = (0,1)$.
2. Justify that f has neither a local maximum nor a local minimum at the point $(x_0,y_0) = (0,1)$.

PROBLEM 2

A closed and bounded region M in the (x,y) plane is delimited by a circle given by the parametric representation

$$\mathbf{r}(u) = \big(\cos(u), \sin(u)\big)\,,\; u \in [0,\, 2\pi]\,.$$

We consider the function

$$f(x,y) = x^2 + y^2 + x\,.$$

1. Determine all stationary points of f in the interior of M.
2. Determine the expression of the composite function

$$f\big(\mathbf{r}(u)\big)\,,\; u \in [0,\, 2\pi]\,.$$

3. Determine the global minimum and the global maximum of f on M.

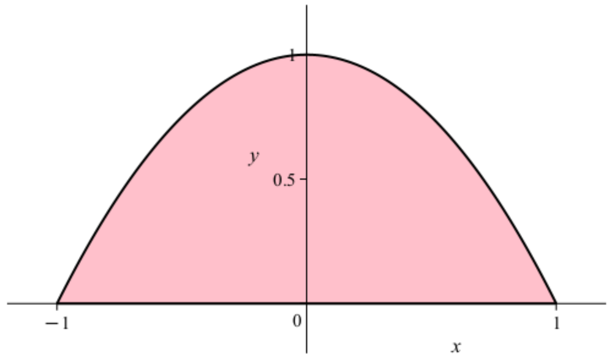
The problem sheet continues on the next page

PROBLEM 3

A region A in the (x,y) plane is given by

$$A = \left\{(x,y) \mid -1 \le x \le 1 \text{ and } 0 \le y \le 1 - x^2\right\},$$

see the figure.



Furthermore, a surface F is given by the part of the graph of the function

$$h(x,y) = 1 - y - x^2$$

that fulfills $y \ge 0$ and $z \ge 0$.

a) Provide a parametric representation of A.
Provide a parametric representation of F. *Hint*: Find your own parametric representation of F, or justify that

$$\mathbf{r}(u,v) = \left(u,\, v(1-u^2), 1 - v(1-u^2) - u^2\right),\ u \in [-1,1],\ v \in [0,1]$$

is a possible parametric representation of F.

Let B denote the closed, spatial region that is located (vertically) between A and F.

b) Provide a parametric representation of B and state the corresponding Jacobian function B.

c) Compute the volume of B.

PROBLEM 4

We consider the solid sphere K in (x,y,z) space that is centred at $(0,0,0)$ and has a radius of 1.

1. State the volume of K.

We are given two vector fields by, respectively,

$$\mathbf{U}(x,y,z) = (x,\, y,\, z) \text{ and } \mathbf{V}(x,y,z) = (x^2 \cdot y,\, -x \cdot y^2,\, z^2).$$

b) Compute the divergence of $\mathbf{U}$ and of $\mathbf{V}$.

c) Compute the flux of $\mathbf{U}$ as well as of $\mathbf{V}$ out through the surface of K.

`End of the problem sheet.`