THE TECHNICAL UNIVERSITY OF DENMARK

Written 2-hour test in the Spring syllabus, May 11 2023.

Course title: Advanced Engineering Mathematics 1. **Course no.:** 01006

Permitted aids: You may bring and use all by DTU permitted aids.

Weight: The four problems weigh equally.

All answers must be well-reasoned, and relevant calculations must be shown to an appropriate extent. You are not allowed to communicate with anyone during the test, not directly nor electronically.

PROBLEM 1

A function $f:\mathbb{R}^2\to\mathbb{R}$ is given by the expression:

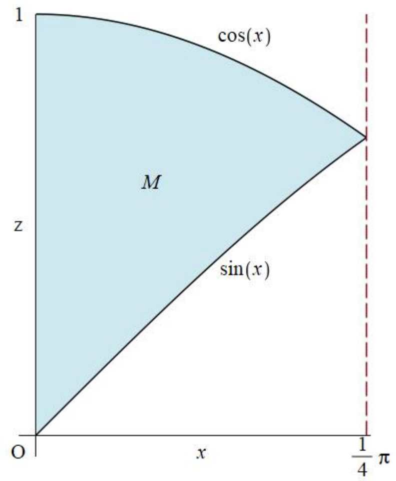
$$f(x,y)=x^2\cdot y-3\cdot x^2-4\cdot y^2+8\cdot y+11\ .$$

1. f has three stationary points. State them.
2. Determine the points in the (x,y) plane at which f has local maximum or local minimum.
3. The parabola with the equation $y=\frac{1}{4}\cdot x^2-1$ along with the straight line with the equation $y=3$ outline a bounded and closed set of points M. Determine the global maximum and the global minimum of f on M, and state the points at which these are found.

PROBLEM 2

A region M in the (x,z) plane is given by

$$M=\{\,(x,z)\,|\,0\leq x\leq\frac{1}{4}\pi\ \text{ and }\ \sin(x)\leq z\leq\cos(x)\,\}\,.$$



1. Provide a parametric representation of M, and compute the area of M.

The problem sheet continues ⟶

A solid of revolution Ω has the parametric representation

$$\mathbf{r}(u,v,w) = \begin{bmatrix} u\cdot\cos(w) \\ u\cdot\sin(w) \\ \sin(u)+v\cdot(\cos(u)-\sin(u)) \end{bmatrix}, \; u\in\left[0,\frac{1}{4}\pi\right], \; v\in[0,1], \; w\in[0,\pi].$$

2. State the Jacobian function that corresponds to $\mathbf{r}$, and compute the volume of Ω.

PROBLEM 3

In the (x,y) plane we are given the velocity vector field $\mathbf{V}(x,y) = \begin{bmatrix} \frac{1}{2}x-\frac{1}{2}y \\ -\frac{1}{2}x+\frac{1}{2}y \end{bmatrix}$.

1. Determine the flow curve $\mathbf{r}(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$ of $\mathbf{V}$ that fulfills the initial condition

$$\mathbf{r}(0) = \begin{bmatrix} x(0) \\ y(0) \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix},$$

and state the point $\mathbf{r}(\ln(3))$.

A curve K has the parametric representation $\mathbf{s}(u) = \begin{bmatrix} u \\ u^2 \end{bmatrix}$, $u\in[0,2]$. At time $t=0$ the curve K starts flowing with $\mathbf{V}$.

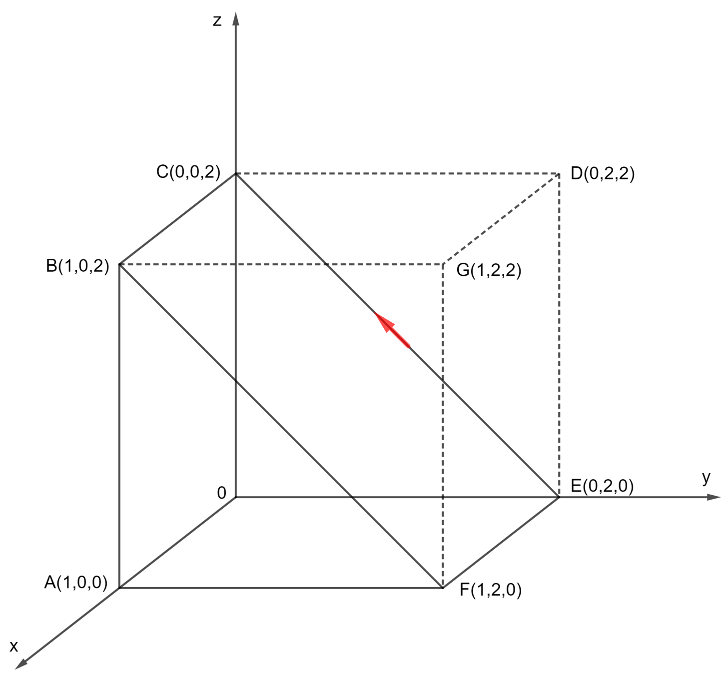
2. State a parametric representation of the curve that K has transformed into at time $t=\ln(3)$.

PROBLEM 4

A vector field $\mathbf{V}$ in (x,y,z) space is given by

$$\mathbf{V}(x,y,z) = (-y\cdot x, y^2+5, -y\cdot z+5\cdot z).$$

A prism P with the corners O,A,B,C,E and F appears when a massive box with the corners O,A,B,C,D,E,F and G is split in two pieces by the plane with the equation $z=2-y$, after which the upper piece (containing the points D and G) is discarded. See the figure.



`The problem sheet continues` ⟶

1. Compute the divergence and the curl of $\mathbf{V}$, and state the volume of P.
2. Compute the flux of $\mathbf{V}$ out through the surface ∂P of P.

Let S denote the surface that is the side of P that contains the corners B, C, E and F.

3. Provide a parametric representation of S, and determine the tangential line integral (the circulation) of $\mathbf{V}$ along the boundary curve ∂S of S where ∂S is given an orientation as indicated with the red arrow on the figure.

End of the problem sheet.