# Theme Exercise 6

# Investigation of Architecture Using Integral Calculus

*In this theme exercise we give a geometrical description of some well-known buildings designed by international architects. We consider surfacae areas , volumes and masses etc. We will need integration in one, two and three variables, but before we reach this state we need of course to establish the necessary parametric representations.*

## Problem 1 The Opera in Beijing

Enjoy initially the sight of the Opera in Beijing, designed by Paul Andreu and build in 2007:

About the building one can on the internet read: *It is a curved building, with a total surface area of* 149, 500 *square meters, that emerges like an island at the center of a lake. The titanium shell is in the shape of a super ellipsoid with a maximum span of* 213 *meters, a minimum span of 144 meters and a height of* 46 *meters).* [ 1 ]

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In the following we assume that the opera building can be modeled as a standard half-ellipsoid.

a) Determine the area of the base of the opera together with its volume using the given information about its maximum and minimum diameter in the base and the height.

b) Assess the information about the total surface area of the building.

c) We assume that the surface shell weighs 20 kg pr. $m^2$ at the bottom and that it decreases continuously with a fixed percentage per unit height until it at the top weighs 10 kg pr. $m^2$. Determine the mass of the surface shell.

## |||| Problem 2 Tycho Brahe Planetarium

Then we are back in Copenhagen and look at the Thyco Brahe Planetarium, designed by the Danish architect Knud Munk and built 1988-89. You notice the beautiful oriental-inspired patterns on the facade, but this is outside the scope of our problem.

The planetarium appears as a cylinder of revolution cut off by an inclined plane. Thereby, the roof forms an ellipse – perhaps a reference to the planets' orbits around the sun. From Wikipedia it appears that the building is 38 meters high, it must be where it is

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highest. From photos we can guess that it is half as high where it is lowest. A selection of the building drawings [2] is available on the Art Library's website. If you look closely at them you will find the radius of the cylinder 12.8 meters and the thickness of the outer wall 0.30 meters.

a) Establish a parametric representation for the planetarium's roof, and determine from that the area of that roof. Hint: The roof can be perceived as a *graph surface* lying over a circular disk in the $(x, y)$ plane with the center at the origin.

b) Determine a parameter representation for the outer wall of the planetarium, and determine the volume of the outer wall.

c) We assume that the stones in the outer wall are massive at the bottom and more porous upwards. Thereby, the density of the stone of the outer wall is 2.3 kg / l at the ground, but it decreases linearly so that it is 1.8 kg / l at the top of the building. Determine the mass of the total outer wall.

## |||| Problem 3 St. Mary Axe in London

Now we move to London and consider Norman Forster's glass-skyscraber St. Mary Axe (popularly known as The Gherkin) from 2004:

Here we shall investigate some of the properties of the building, based on a parametrization of its surface, that is a surface of revolution. The building is thought to be imbedded in an ordinary $(x, y, z)$-coordinate system such that the axis of revolution

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of the building runs along the $z$-axis. We assume that the profile curve, placed in the $(x, z)$-plane, can be modeled by a function of the form

$$x = f(z) = \sqrt{az^2 + bz + c} \text{ hvor } a,\, b,\, c \in \mathbb{R}\,.$$

a) Determine $a$, $b$ and $c$ from the following information that can be found on the internet, see e.g. [3] and [4]: 1) The building is 180 meters heigh, 2) radius at the bottom is 24 meters and 3) the building reaches its maximum width at the height 66 meters.

b) State a parametric representation for the surface of the building and illustrate.

c) Determine the area of the surface.

It is given in the said material that in total 8358 tons of steel has been used in the building. Let $m(z)$, $z \in [0, 180]$, denote the average mass density of steel in the building at the height $z$, measured in $tons/m^3$. It is assumed that $m(z)$ decreases linearly with the height such that it at the top is half as large as it is at the bottom.

d) Determine $m(0)$.

*The Theme Exercise Ends*